Published in Phys. Lett. B376 (1996) 19.
ADP-95-50/T197
hep-ph/9601309

Analysis of rho-omega interference in the pion form-factor

Kim Maltman[a,b], H.B. O'Connell[b] and A.G. Williams[b,c]

[a]*Department of Mathematics and Statistics, York University, 4700 Keele St., North York, Ontario, Canada M3J 1P3*

[b]*Department of Physics and Mathematical Physics, University of Adelaide 5005, Australia*

[c]*Institute for Theoretical Physics, University of Adelaide 5005, Australia*

14 February, 1996

Abstract

The formalism underlying the analysis of $e^+e^- \to \pi^+\pi^-$ in the $\rho - \omega$ interference region is carefully revisited. We show that the standard neglect of the pure $I = 0$ omega, ω_I, "direct" coupling to $\pi\pi$ is not valid, and extract those combinations of the direct coupling and ρ-ω mixing allowed by experiment. The latter is shown to be only very weakly constrained by experiment, and we conclude that data from the $e^+e^- \to \pi^+\pi^-$ interference region *cannot* be used to fix the value of ρ–ω mixing in a model-independent way unless the errors on the experimental phase can be significantly re-

duced. Certain other modifications of the usual formalism necessitated by the unavoidable momentum-dependence of $\rho-\omega$ mixing are also discussed.

E-mail: *hoconnel, awilliam@physics.adelaide.edu.au; FS300175@sol.yorku.ca*

Keywords: vector mesons, pion form-factor, mixing, isospin.

PACS: 11.20.Fm, 12.40.Vv, 13.60.Le, 13.65.+i

The cross-section for $e^+e^- \to \pi^+\pi^-$ in the $\rho{-}\omega$ resonance region displays a narrow interference shoulder resulting from the superposition of narrow resonant ω and broad resonant ρ exchange amplitudes [1]. The strength of the ω "interference" amplitude has generally been taken to provide a measurement of ρ_I-ω_I mixing (where ρ_I, ω_I are the pure isovector ρ and isoscalar ω states) [2,3]. The extracted mixing has then been used to generate ρ_I-ω_I mixing contributions to various few-body observables [4–6], a program which, combined with estimates for other sources of isospin-breaking, produces predictions for few-body isospin breaking in satisfactory accord with experiment [5]. The phenomenological success, for those observables for which ρ_I-ω_I contributions are significant, rests, inextricably, on two assumptions, (1) that the interference amplitude is dominated by ρ_I-ω_I mixing (i.e., negligible "direct" $\omega_I \to \pi\pi$ contribution to the physical ω decay amplitude) and (2) that the resulting mixing amplitude is independent of momentum-squared, so the extracted value can be used unchanged in meson-exchange forces in few-body systems, where $q^2 < 0$.

The neglect of "direct" $\omega_I \to \pi\pi$ coupling (i.e., coupling which does not go via mixing with the ρ_I) can actually be re-interpreted physically, this re-interpretation simultaneously providing the conventional justification for taking the ρ_I-ω_I self-energy, $\Pi^{\rho\omega}$, to be real in modern analyses of $e^+e^- \to \pi^+\pi^-$ [7,8]. As will become clear below, however, corrections to the underlying argument, usually thought to be small, have unexpectedly large effects on the extraction of the $\rho{-}\omega$ mixing contribution from experimental data.

The assumption of the q^2-independence of $\Pi^{\rho\omega}(q^2)$ is more problematic [9,10]. In general, one knows that a system of, e.g., nucleons, vector mesons and pseudoscalar mesons, can be described by an effective low-energy Lagrangian, constructed so as to be compatible with QCD (e.g., one might think of the effective chiral Lagrangian, $\mathcal{L}_{\rm eff}$, obtainable via the Coleman-Callan-Wess-Zumino construction [11]). Such a Lagrangian, involving terms of arbitrarily high order in derivatives, will produce momentum-dependence in all

observables which can in principle become momentum-dependent. This has been seen explicitly for the off-diagonal (mixing) elements of meson propagators by a number of authors, employing various models [12,13], as well as QCD sum rule and Chiral Perturbation Theory (ChPT) techniques [14]. Such q^2-dependence has also been shown to be consistent with the usual vector meson dominance (VMD) framework [15]. The possibility [16] that an alternative choice of interpolating fields might, nonetheless, correspond to the standard assumption of q^2-independence has been shown to be incompatible with the constraints of unitarity and analyticity [17]. It is thus appropriate to revisit and generalize the usual analysis.

As has been known for some time, to obtain properties of unstable particles which are process-independent and physically meaningful, one determines the locations of the resonance poles in the amplitude under consideration, and makes expansions about these pole locations [18]. The (complex) pole locations are properties of the S-matrix and hence *independent of the choice of interpolating fields*, and the separate terms in the Laurent expansion about the pole position have well-defined physical meaning [18]. The importance of such an "S-matrix" formalism for characterizing resonance properties has been stressed recently by a number of authors in the context of providing gauge- and process-independent definitions of the Z^0 mass and width in the Standard Model [19,20]. For our purposes this means that: (1) the "physical" $\{\rho,\, \omega\}$ fields are to be identified as those combinations of the $\{\rho_I,\, \omega_I\}$ fields containing the corresponding S-matrix poles and (2) to analyze $e^+e^- \to \pi^+\pi^-$ one should include both resonant terms involving the complex ρ and ω pole locations (and hence constant widths) and "background" (i.e. non-resonant) terms. In quoting experimental results we will, therefore, restrict ourselves to analyses which, as closely as possible, satisfy these requirements. To our knowledge, only one such exists: the fifth fit of Ref. [21] (performed explicitly in the S-matrix formalism, though without an s-dependence to the background). As stressed in Ref. [21], using the S-

matrix formalism, one finds a somewhat lower real part for the (complex) ρ pole position ($\hat{m}_\rho = 757.00 \pm 0.59,\ \Gamma_\rho = 143.41 \pm 1.27$ MeV) than is obtained in conventional, non-S-matrix formalism treatments. For comparison below we will also employ the results of the second fit of the more conventional (but non-S-matrix) formalism of Ref. [22], which employs an s-dependent background, an s-dependent ρ width, and imposes the (likely too large) Particle Data Group value for the ρ mass by hand.

Let us turn to the question of $\rho-\omega$ mixing in the presence of a q^2-dependent off-diagonal element of the self-energy matrix. We shall work consistently to first order in isospin breaking (generically, $\mathcal{O}(\epsilon)$), which will mean to first order in $\Pi_{\rho\omega}$. The dressing of the bare, two-channel meson propagator has been treated in Ref. [10].

As we consider vector mesons coupled to conserved currents, we can replace $D_{\mu\nu}(q^2)$ by $-g_{\mu\nu}D(q^2)$. We refer to $D(q^2)$ as the "scalar propagator". We assume that the isospin-pure fields ρ_I and ω_I have already been renormalized, i.e., that the relevant counterterms have been absorbed into the mass and wavefunction renormalizations. Taking then the full expression for the dressed propagator and keeping terms to $\mathcal{O}(\epsilon)$, one finds

$$D^I(q^2) = \begin{pmatrix} D^I_{\rho\rho} & D^I_{\rho\omega} \\ D^I_{\rho\omega} & D^I_{\omega\omega} \end{pmatrix} = \begin{pmatrix} (q^2 - \Pi_{\rho\rho}(q^2))^{-1} & D^I_{\rho\omega}(q^2) \\ D^I_{\rho\omega}(q^2) & (q^2 - \Pi_{\omega\omega}(q^2))^{-1} \end{pmatrix}, \tag{1}$$

where the renormalized self-energies $\Pi_{kk}(q^2) \to m_k^2$ as $q^2 \to m_k^2$. Defining $\Pi^{(0)}_{kk}(q^2) = \Pi_{kk}(q^2) - m_k^2$, we then have $\Pi^{(0)}_{kk}(q^2) = \mathcal{O}[(q^2 - m_k^2)^2]$. From the complex pole positions, m_k^2, we define the (real) mass ($\hat{m}_k$) and width (Γ_k) via, $m_k^2 \equiv \hat{m}_k^2 - i\,\hat{m}_k\Gamma_k$. To $\mathcal{O}(\epsilon)$, $D^I_{\rho\omega}(q^2)$, is then [10]

$$D^I_{\rho\omega}(q^2) = \frac{\Pi_{\rho\omega}(q^2)}{(q^2 - m_\rho^2 - \Pi^{(0)}_{\rho\rho}(q^2))(q^2 - m_\omega^2 - \Pi^{(0)}_{\omega\omega}(q^2))} = D^I_{\rho\rho}(q^2)\Pi_{\rho\omega}(q^2)D^I_{\omega\omega}(q^2), \tag{2}$$

which contains both a broad ρ resonance and narrow ω resonance piece.

As explained above, the physical ρ and ω fields are defined to be those combinations of the ρ_I and ω_I for which only the diagonal elements of the propagator matrix contain poles,

in the ρ, ω basis. This definition is, in fact, implicit in the standard interpretation of the $e^+e^- \to \pi^+\pi^-$ experiment, which associates the broad resonant part of the full amplitude with the ρ and the narrow resonant part with the ω. Using different linear combinations of ρ_I, ω_I, (call them ρ', ω') than those given above (ρ, ω), one would find also narrow resonant structure in the off-diagonal element of the vector meson propagator in the $\{\rho'$, $\omega'\}$ basis, preventing, for example, the association of the narrow resonant behaviour with the ω' pole term alone.

We define the transformation between the physical and isospin pure bases by (to $\mathcal{O}(\epsilon)$)

$$\rho = \rho_I - \epsilon_1\, \omega_I, \qquad \omega = \omega_I + \epsilon_2\, \rho_I \tag{3}$$

where, in general, $\epsilon_1 \neq \epsilon_2$ when the mixing is q^2-dependent. With $D^{\mu\nu}_{\rho\omega}(x-y) \equiv -i\langle 0|T(\rho^\mu(x)\omega^\nu(y))|0\rangle$, one then has for the scalar propagator, to $\mathcal{O}(\epsilon)$,

$$D_{\rho\omega}(q^2) = D^I_{\rho\omega}(q^2) - \epsilon_1 D^I_{\omega\omega}(q^2) + \epsilon_2 D^I_{\rho\rho}(q^2). \tag{4}$$

The condition that $D_{\rho\omega}(q^2)$ contain no ρ or ω pole then fixes $\epsilon_{1,2}$ to be

$$\epsilon_1 = \frac{\Pi_{\rho\omega}(m_\omega^2)}{m_\omega^2 - m_\rho^2 - \Pi^{(0)}_{\rho\rho}(m_\omega^2)}, \qquad \epsilon_2 = \frac{\Pi_{\rho\omega}(m_\rho^2)}{m_\omega^2 - m_\rho^2 + \Pi^{(0)}_{\omega\omega}(m_\rho^2)}\ . \tag{5}$$

When $\Pi^{\rho\omega}(q^2)$ is q^2-dependent, we thus see explicitly that $\epsilon_1 \neq \epsilon_2$; the relation between the isospin-pure and physical bases is not a simple rotation. This is a universal feature of q^2-dependent mixing in field theory. Recall that $\Pi^{(0)}_{\rho\rho}(q^2)$ and $\Pi^{(0)}_{\omega\omega}(q^2)$ vanish by definition as $q^2 \to m^2_{\rho,\omega}$ at least as fast as $(q^2 - m^2_{\rho,\omega})^2$. The usual assumption is that these two quantities are zero in the vicinity of the resonance region, which leads to the standard Breit-Wigner form for the vector meson propagators. $\Pi^{(0)}_{\rho\rho}(q^2)$ and $\Pi^{(0)}_{\omega\omega}(q^2)$ are, of course, momentum-dependent in general since the vector propagators must be real below the $\pi\pi$ and $\pi\gamma$ thresholds. Note that, from Eqs. (4) and (5), any deviation from the Breit-Wigner form and/or any non-linearity in the q^2-dependence of $\Pi_{\rho\omega}(q^2)$ will produce a non-zero off-diagonal element of the vector propagator *even in the physical basis.* This means

that a background (non-resonant) term is completely unavoidable even in the traditional VMD framework, where all contributions are associated with vector meson exchange. Moreover, in general, this background will be s- (i.e., q^2)-dependent. Finally, even in the vicinity of the ρ and ω poles, where it should be reasonable to set $\Pi^{(0)}_{\rho\rho}(q^2)$ and $\Pi^{(0)}_{\omega\omega}(q^2)$ to zero, the ρ_I admixture into the physical ω is governed, not by $\Pi^{\rho\omega}(m^2_\omega)$ as usually assumed, but by $\Pi^{\rho\omega}(m^2_\rho)$.

The time-like EM pion form-factor is given, in the interference region, by

$$F_\pi(q^2) = \left[g_{\omega\pi\pi} D_{\omega\omega} \frac{f_{\omega\gamma}}{e} + g_{\rho\pi\pi} D_{\rho\rho} \frac{f_{\rho\gamma}}{e} + g_{\rho\pi\pi} D_{\rho\omega} \frac{f_{\omega\gamma}}{e} \right] + \text{background}, \tag{6}$$

where $g_{\omega\pi\pi}$ is the coupling of the *physical* omega to the two pion final state and $f_{\rho\gamma}$ and $f_{\omega\gamma}$ are the electromagnetic ρ and ω couplings. The third piece of Eq. (6), $g_{\rho\pi\pi} D_{\rho\omega} f_{\omega\gamma}$, results from the non-vanishing of the off-diagonal element of the *physical* meson propagator and, being non-resonant, can be absorbed into the background, for the purposes of our discussion, as can any deviations from the Breit-Wigner form for the ρ and ω propagators. Since the variation of q^2 over the interference region is tiny, we can presumably also safely neglect any q^2-dependence of $f_{\rho\gamma}$, $f_{\omega\gamma}$, $g_{\rho\pi\pi}$ and $g_{\omega\pi\pi}$. $f_{V\gamma}$ is related to the "universality coupling" [15], g_V, of traditional VMD treatments by $f_{V\gamma} = -e\hat{m}^2/g_V$.

We now focus on the resonant ω exchange contribution, whose magnitude and phase, relative to the resonant ρ exchange, are extracted experimentally. We have

$$g_{\omega\pi\pi} = \langle \pi\pi | \omega_I + \epsilon_2 \rho_I \rangle = g_{\omega_I\pi\pi} + \epsilon_2 g_{\rho_I\pi\pi}, \tag{7}$$

where ϵ_2 is given in Eq. (5) or, equivalently, by $\epsilon_2 = -i\, z \Pi_{\rho\omega}(m^2_\rho)/\hat{m}_\rho \Gamma_\rho$, where

$$z \equiv \left[1 - \frac{\hat{m}_\omega \Gamma_\omega}{\hat{m}_\rho \Gamma_\rho} - i \left(\frac{\hat{m}^2_\omega - \hat{m}^2_\rho}{\hat{m}_\rho \Gamma_\rho} \right) \right]^{-1} . \tag{8}$$

Note that $z \approx 1$ but equals 1 only if we neglect the ω width and $\rho - \omega$ mass difference. This brings us to the Renard argument [7]. Since, in general, $g_{\omega_I\pi\pi} \neq 0$, $\Pi_{\rho\omega}(q^2)$ must

contain a contribution from the intermediate $\pi\pi$ state which, because essentially the entire ρ width is due to the $\pi\pi$ mode, is given by

$$\Pi^{2\pi}_{\rho\omega}(m_\rho^2) = \frac{g_{\omega_I\pi\pi}}{g_{\rho_I\pi\pi}}\Pi^{2\pi}_{\rho\rho}(m_\rho^2) = G(\mathrm{Re}\Pi^{2\pi}_{\rho\rho}(m_\rho^2) - i\hat{m}_\rho\Gamma_\rho), \tag{9}$$

where $G = g_{\omega_I\pi\pi}/g_{\rho_I\pi\pi}$ is the ratio of the ρ_I and ω_I couplings to $\pi\pi$. In arriving at Eq. 9 we have used the facts that (1) the imaginary part of the ρ self-energy at resonance ($q^2 = m_\rho^2$) is, by definition, $-\hat{m}_\rho\Gamma_\rho$, and (2) $g_{\rho\pi\pi} = g_{\rho_I\pi\pi}$ to $\mathcal{O}(\epsilon)$. We have then, defining $\tilde{\Pi}_{\rho\omega}$ by $\Pi_{\rho\omega} = \tilde{\Pi}_{\rho\omega} - iG\hat{m}\Gamma_\rho$,

$$\epsilon_2 = z\frac{-i}{\hat{m}_\rho\Gamma_\rho}[\tilde{\Pi}_{\rho\omega}(m_\rho^2) - iG\hat{m}_\rho\Gamma_\rho] \tag{10}$$

and hence

$$g_{\omega\pi\pi} = g_{\omega_I\pi\pi}\,(1-z) + \tilde{\epsilon}_2 g_{\rho_I\pi\pi}, \tag{11}$$

where $\tilde{\epsilon}_2 = (-iz/\hat{m}_\rho\Gamma_\rho)\tilde{\Pi}_{\rho\omega}(m_\rho^2)$. We shall also define, for convenience,

$$\tilde{T} \equiv \tilde{\Pi}_{\rho\omega}(m_\rho^2)/\hat{m}_\rho\Gamma_\rho. \tag{12}$$

The standard Renard analysis [7] involves approximating z by 1. The contribution to $\omega \to \pi\pi$ from the intrinsic ω_I decay is then exactly cancelled in Eq. (11). Using the (preferred) experimental analysis of Ref. [21], however, we find

$$z = 0.9324 + 0.3511\,i\ . \tag{13}$$

(For comparison, the analysis of Ref. [22] gives $1.023{+}0.2038i$). Because of the substantial imaginary part, the intrinsic decay cannot be neglected in $e^+e^- \to \pi^+\pi^-$.

Substituting the results above into Eq. (6), we find

$$F_\pi(q^2) = \frac{f_{\rho\gamma}}{e}g_{\rho_I\pi\pi}\left[|r_{\mathrm{ex}}|e^{i\phi_{e^+e^-}}\left((1-z)G - \mathrm{i}z\tilde{T}\right)P_\omega + P_\rho\right] + \text{ background}, \tag{14}$$

where we have replaced the propagators $D_{\rho\rho,\omega\omega}$ of Eq. (6) with the simple Breit-Wigner pole terms $P_{\rho,\omega} \equiv 1/(p^2 - m^2_{\rho,\omega})$, and where

$$r_{\rm ex} \equiv \frac{f_{\omega\gamma}}{f_{\rho\gamma}} = |r_{\rm ex}| e^{{\rm i}\phi_{e^+e^-}}, \tag{15}$$

with $\phi_{e^+e^-}$ the "leptonic phase" (to be discussed in more detail below). Experimentally,

$$|r_{\rm ex}| = \left[\frac{\hat{m}_\omega^3 \Gamma(\omega \to e^+e^-)}{\hat{m}_\rho^3 \Gamma(\rho \to e^+e^-)}\right]^{1/2} = 0.30 \pm 0.01 \tag{16}$$

using the values found in Ref. [21]. The form of $F_\pi(q^2)$ in Eq. (14) is what is required for comparison with experimental data [21], for which one has

$$F_\pi \propto P_\rho + A e^{i\phi} P_\omega; \quad A = -0.0109 \pm 0.0011; \quad \phi = (116.7 \pm 5.8)^{\rm o}. \tag{17}$$

One can now see that the uncertainty in the Orsay phase, ϕ, makes a precise extraction of $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$ impossible. Indeed, the two contributions to the ω exchange amplitude (i.e., multiplying P_ω) have either nearly the same phase or differ in phase by close to π (depending on the relative signs of G and $\tilde{T}$). In either case, a large range of combinations of G and $\tilde{T}$, all producing nearly the same overall phase, will produce the same value of A. The experimental data can thus place only rather weak constraints on the relative size of the two contributions, as we will see more quantitatively below.

Let us write $r_{\rm ex}$, the ratio of electromagnetic couplings, in terms of the corresponding isospin-pure ratio, $r_I = f_{\omega_I\gamma}/f_{\rho_I\gamma}$. Using $f_{\omega\gamma} = f_{\omega_I\gamma} + \epsilon_2 f_{\rho_I\gamma}$ and $f_{\rho\gamma} = f_{\rho_I\gamma} - \epsilon_1 f_{\omega_I\gamma}$, one finds $r_{\rm ex} = (r_I + \epsilon_2)/(1 - \epsilon_1 r_I)$, where r_I is real. To $\mathcal{O}(\epsilon)$ one then has

$$\sin\phi_{e^+e^-} = \frac{{\rm Im}(\epsilon_2) + |r_{\rm ex}|^2 {\rm Im}(\epsilon_1)}{|r_{\rm ex}|}. \tag{18}$$

Ignoring the small difference in ϵ_1 and ϵ_2 (since $r_{\rm ex}^2$ is small) we obtain

$$\sin\phi_{e^+e^-} = \frac{(1 + |r_{\rm ex}|^2){\rm Im}\epsilon_2}{|r_{\rm ex}|}. \tag{19}$$

In order to simplify the discussion of our main point, which is the effect of including the direct coupling on the experimental analysis, let us now make the usual assumption that the imaginary part of $\Pi_{\rho\omega}$ is dominated by $\pi\pi$ intermediate states. (Note, however,

that, because the argument is complex, there may be an imaginary part of $\Pi_{\rho\omega}$ even in the absence of real intermediate states; for example, in the model of Ref. [13], with confined quark propagators, the phase of the quark loop contribution to $\Pi_{\rho\omega}(m_\rho^2)$ is about $-13^{\rm o}$ [23], despite the model having, for this contribution, no available intermediate states.) Making this assumption, $\tilde{\Pi}_{\rho\omega}$ (and thus $\tilde{T}$) becomes pure real and the imaginary part of $\Pi_{\rho\omega}(m_\rho^2)$ reduces to $-G\hat{m}_\rho\Gamma_\rho$. Using Eqs. (10) and (19) the leptonic phase becomes

$$\sin\phi_{e^+e^-} = -\left(\frac{1+|r_{\rm ex}|^2}{|r_{\rm ex}|}\right)(\tilde{T}\,{\rm Re}\,z + G\,{\rm Im}\,z) \tag{20}$$

which is completely fixed by G and $\tilde{\Pi}_{\rho\omega}$. For each possible $\tilde{\Pi}_{\rho\omega}$, only one solution for G both gives the correct experimental magnitude for the ω exchange amplitude (A) and has a phase lying in the second quadrant, as required by experiment. Knowing $\tilde{\Pi}_{\rho\omega}$ and G, Eqn. (20) allows us to compute the total phase, ϕ. Those pairs $(\tilde{\Pi}_{\rho\omega}, G)$ producing the experimentally allowed (A, ϕ) constitute our full solution set.

The results of the above analysis are presented in Fig. 1, where we have used as input the results of the analysis of Ref. [21], for the reasons explained above. The spread in G values reflects the experimental error in A. We will supplement the experimental constraints by imposing the theoretical prejudice $-0.05 < G < 0.05$. We see that, barring theoretical input on the precise size of G, experimental data is incapable of providing even reasonably precise constraints on the individual magnitudes of G and $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$. The reason for this situation has been explained above. If we fix A at its central value, the experimental phase alone would restrict $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$ to the range (-1090 MeV^2, -5980 MeV^2), the G constraint to the range (-2290 MeV^2, -6180 MeV^2). Including the experimental error on A extends, for example, the phase constraint range to (-840 MeV^2, -6240 MeV^2). For comparison, artificially setting $G = 0$ produces $\tilde{\Pi}_{\rho\omega}(m_\rho^2) = -3960$ MeV^2. One may repeat the above analysis using the input parameters of Ref. [22] (where, however, the ρ pole position is presumably high by about 10 MeV [21]). For the central A value, the experimentally allowed range of $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$ is (-3720 MeV^2, -5080 MeV^2). The large

uncertainty in the extracted values of $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$ and G is thus not an artifact of the particular fit of Ref. [21]. The small (± 600 MeV^2) error usually quoted for $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$, and associated with the experimental error in the determination of A, thus represents a highly inaccurate statement of the true uncertainty in the extraction of this quantity from the experimental data. It is important to stress that no further information on $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$ is obtainable from the $e^+e^- \to \pi^+\pi^-$ data without additional theoretical input.

Note that, in the model of Ref. [13], as currently parametrized, the sign of G is determined to be positive, and the magnitude to be $\simeq 0.02$. Such a value of G, however, coupled with the phase correction mentioned above, would fail to satisfy the experimental phase constraint. This shows that, despite the weakness of the experimental constraints for the magnitudes of G and $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$, the experimental results are, nonetheless, still capable of providing non-trivial constraints for models of the mixing.

In conclusion, we have shown that, in general, there is a contribution to the $\rho-\omega$ interference in $e^+e^- \to \pi^+\pi^-$ which arises from the intrinsic $\omega_I \to \pi\pi$ coupling, and that this contribution, given the current level of accuracy of the experimentally extracted Orsay phase, precludes any even reasonably precise extraction of the $\rho-\omega$ mixing in the absence of additional theoretical input. It is important to stress that this conclusion and the central result of Eq. (14) do not depend in the least on the possible q^2-dependence of $\Pi_{\rho\omega}(q^2)$ nor on the use of the S-matrix formalism: even for constant $\Pi_{\rho\omega}$ and a more traditional Breit-Wigner analysis one would still have a significant imaginary part of z and hence a residual contribution from the direct coupling which, being nearly parallel to that associated with $\rho-\omega$ mixing, would lead also to the conclusion stated above. Note, however, that a significant improvement in the determination of the experimental phase would allow one to simultaneously extract the self-energy and the isospin-breaking ratio, G. In addition to the main point, just discussed, we also note that (1) even if G were, for some reason, to be zero, the data would provide the value of the mixing amplitude at

m_ρ^2 and not m_ω^2, (2) since it is the complex S-matrix pole positions of the ρ and ω which govern the mixing parameters $\epsilon_{1,2}$, only an analysis utilizing the S-matrix formalism can provide reliable input for these pole positions, and hence for the analysis of the isospin-breaking interference in $e^+e^- \to \pi^+\pi^-$ and (3) the simultaneous use of the experimental magnitude and phase can provide non-trivial constraints on models of the vector meson mixing process.

Acknowledgements: We would like to thank M. Benayoun for discussions on the experimental situation and K. Mitchell and P. Tandy for providing information about the phase of the $\rho-\omega$ mixing amplitude off the real axis in the model of Ref. [13] prior to publication. We also acknowledge helpful discussions with A. Thomas.

REFERENCES

[1] L.M. Barkov *et al.*, Nucl. Phys. B256 (1985) 365.

[2] S.A. Coon and R.C. Barrett, Phys. Rev. C 36 (1987) 2189 .

[3] H.B. O'Connell, B.C. Pearce, A.W. Thomas and A.G. Williams, Phys. Lett. B354 (1995) 14.

[4] E.M. Henley and G.A. Miller, in *Mesons in Nuclei*, ed. M. Rho and D.H. Wilkinson (North Holland, 1979).

[5] G.A. Miller, B.M.K. Nefkens and I. Šlaus, Phys. Rep. 194 (1990) 1;

[6] G.A. Miller, A.W. Thomas and A.G. Williams, Phys. Rev. Lett. 56 (1986) 2567; A.G. Williams, *et al.*, Phys. Rev. C 36 (1987) 1956; G.A. Miller and W.T.H van Oers, to appear in Symmetries and Fundamental Interactions in Nuclei, eds. E.M. Henley and W.C. Haxton, nucl-th/9409013.

[7] F.M. Renard in Springer Tracts in Modern Physics 63, 98, Springer-Verlag (1972).

[8] H.B. O'Connell, B.C. Pearce, A.W. Thomas, and A.G. Williams, Adelaide U. preprint, hep-ph/9501251.

[9] S. Coleman and H.J. Schnitzer, Phys. Rev. 134 (1964) B863; R.G. Sachs and J.F. Willemsen, Phys. Rev. D 2 (1970) 133.

[10] H.B. O'Connell, B.C. Pearce, A.W. Thomas, and A.G. Williams, Phys. Lett. B336 (1994) 1.

[11] S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177 (1969) 2239; C.G. Callan, S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177 (1969) 2247.

[12] T. Goldman, J.A. Henderson and A.W. Thomas, Few Body Systems 12 (1992)

123; J. Piekarawicz and A.G. Williams, Phys. Rev. C 47 (1993) R2462; G. Krein, A.W. Thomas and A.G. Williams, Phys. Lett. B317 (1993) 293; R. Friedrich and H. Reinhardt, Nucl. Phys. A594 (1995) 406.

[13] K. Mitchell, P. Tandy, C. Roberts and R. Cahill, Phys. Lett. B335 (1994) 282.

[14] T. Hatsuda, E.M. Henley, Th. Meissner and G. Krein, Phys. Rev. C 49 (1994) 452; M.J. Iqbal, X. Jin and D.B. Leinweber, TRIUMF pre-prints, nucl-th/9504026 and nucl-th/9507026; R. Urech, Phys. Lett. B355 (1995) 308; K. Maltman, Phys. Lett. B313 (1993) 203, Phys. Lett. B351 (1995) 56, Phys. Rev. D53 (1996) 2563 and 2573.

[15] H.B. O'Connell, A.G. Williams, M. Bracco, and G. Krein, Phys. Lett. B370 (1996) 12.

[16] T.D. Cohen and G.A. Miller, Phys. Rev. C 52 (1995) 3428.

[17] K. Maltman, Phys. Lett. B362 (1995) 11.

[18] R. Eden, P. Landshoff, D. Olive and J. Polkinghorne, *The Analytic S-matrix*, Cambridge University Press, Cambridge (1966).

[19] A. Sirlin, Phys. Rev. Lett. 67 (1991) 2127.

[20] R.G. Stuart, Phys. Rev. Lett. 70 (1993) 3193, Phys. Rev. D 52 (1995) 1655, hep-ph/9504308; M. Nowakowski and A. Pilaftis, Z. Phys. C 60 (1993) 121; J. Papavassiliou and A. Pilaftis, Phys. Rev. Lett. 75 (1995) 3060; hep-ph/9507246.

[21] A. Bernicha, G. López Castro and J. Pestieau, Phys. Rev. D 50 (1994) 4454, hep-ph/9510435.

[22] M. Benayoun *et al.*, Zeit. Phys. C 58 (1993) 31.

[23] K. Mitchell and P. Tandy, private communication.

FIGURES

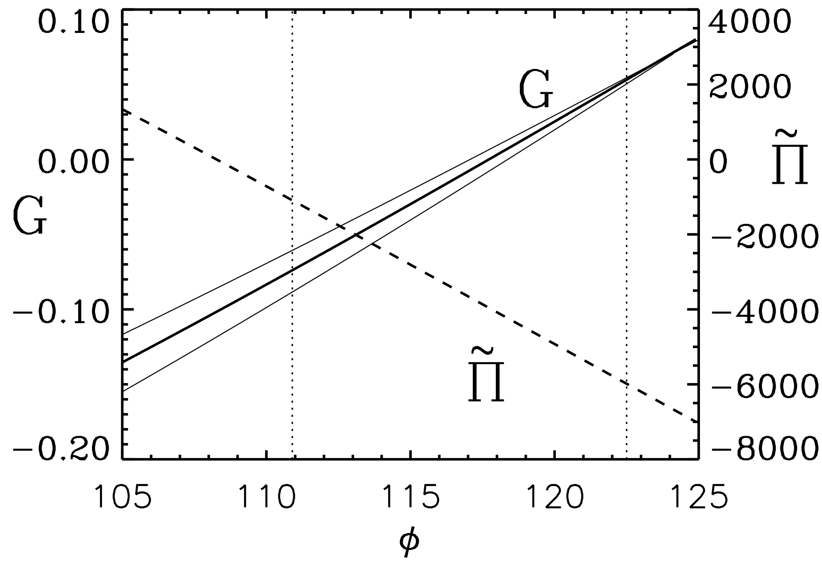

FIG. 1. The allowed values of $G = g_{\omega_I\pi\pi}/g_{\rho_I\pi\pi}$ and $\tilde{\Pi}(m_\rho^2)$ (in MeV^2) are plotted as a function of the Orsay phase, ϕ. The vertical lines indicate the experimental uncertainty in ϕ $(= 116.7 \pm 5.8)^{\mathrm{o}}$ and the uncertainty in the amplitude A (0.0109 ± 0.0011) (see text) gives rise to the spread of possible values of G at each value of ϕ.